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ORIGINAL PAPER

Comment on "The best Condorcet-compatible election method: Ranked Pairs"

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Abstract
In the paper "The best Condorcet-compatible election method: Ranked Pairs" (Munger in Const Polit Econ 2023. https://doi.org/10.1007/s10602-022-09382-w), the author identifies Tideman's ranked pairs method as the best single-winner voting method, while the Schulze method (Schulze in Soc Choice Welf 36(2):267–303, 2011; The Schulze method of voting, 2018. https://arxiv.org/pdf/1804.02973v12.pdf) comes in a close second. Munger (Munger in Const Polit Econ 2023. https://doi.org/10.1007/s10602-022-09382-w) writes: "Beatpath (Schulze) emerges as Ranked Pair's chief competitor". In this comment, I will show that the scenario that Munger uses to argue against the Schulze method is not possible.

Keywords Ranked pairs · Schulze · Beatpath · Kemeny–Young · Ranked choice ballots

JEL Classification C02 · D71 · D72 · K16 · H11

1 Schulze method

The Schulze method is defined as follows:

- $N[x,y]$ is the number of voters who prefer x to y.
- $N[x,y] - N[y,x]$ is the *strength* of the link xy
- A *path* from x to y is a sequence of candidates $c(1),\ldots,c(n)$ that starts with $x=c(1)$ and ends with $y=c(n)$ and that visits the same candidate at most once.
- The *strength* of a path is the strength of its weakest link. So the *strength* of the path $c(1),\ldots,c(n)$ is the minimum of $N[c(i),c(i+1)] - N[c(i+1),c(i)]$ for $i=1,\ldots,(n-1)$.

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- $P[x,y]$ is the strength of the strongest path from x to y.
- x *disqualifies* y if and only if $P[x,y] > P[y,x]$.
- x is a *winner* if and only if $P[x,y] \geq P[y,x]$ for every other y.

The following facts can be proven:

- If $P[x,y] > P[y,x]$ and $P[y,z] > P[z,y]$, then $P[x,z] > P[z,x]$.
- There is always at least one winner.
- x is a unique winner if and only if $P[x,y] > P[y,x]$ for every other y.

For precise definitions and proofs, I refer the reader to Schulze (2018).

2 Munger's scenario

In Munger's scenario, a is identified as best candidate and b is identified as second best candidate; but when a is removed then b is identified as worst candidate. Munger (2023) writes: "If the order of candidates is to have something to do with merit, then if the candidate who placed first had not run, then the candidate who placed second should have won. Kemeny–Young and Ranked Pairs have that property, but Beatpath (Schulze) fails it in the worst possible way; the candidate who placed second can become ranked *last* of the remaining candidates." However, I will now prove that Munger's scenario is not possible under the Schulze method.

2.1 Step #1

Suppose A is the set of candidates.

Suppose b is the second best candidate in the presence of a, then we get:

$$\text{For every } x \in A \backslash \{a,b\} : P^{\text{old}}[b,x] > P^{\text{old}}[x,b]. \tag{1}$$

Suppose b is the worst candidate in the absence of a, then we get:

$$\text{For every } x \in A \backslash \{a,b\} : P^{\text{new}}[x,b] > P^{\text{new}}[b,x]. \tag{2}$$

When a candidate is removed, then the strength of the strongest path between two other candidates x and y either decreases or doesn't change. But it cannot increase, since the set of possible paths between x and y is reduced. Therefore, we get:

$$\text{For every } x \in A \backslash \{a,b\} : P^{\text{old}}[x,b] \geq P^{\text{new}}[x,b]. \tag{3}$$

With (1), (2) and (3), we get:

$$\text{For every } x \in A \backslash \{a,b\} : P^{\text{old}}[b,x] > P^{\text{new}}[b,x]. \tag{4}$$

But $P^{\text{old}}[b,x] > P^{\text{new}}[b,x]$ is possible only when a was in the strongest path from b to x. Therefore with (4), we get:

For every $x \in A \backslash \{a,b\}$: In the presence of a, a is in the strongest path from b to x. (5)

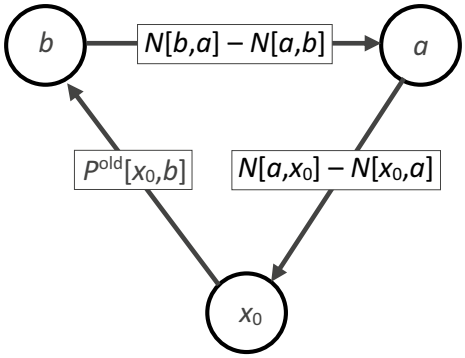
2.2 Step #2

Suppose there are candidates $x_1, x_2 \in A \setminus \{a,b\}$, such that bx_1 is the first link in the strongest path from b to x_2. Then the link bx_1 is already a strongest path from b to x_1 that doesn't contain a. But this is a contradiction to (5). Therefore, we get:

For every $x \in A \backslash \{a,b\}$: In the presence of a, ba is the first link in the strongest path from b to x. (6)

2.3 Step #3

Suppose ax_0 is the first link in the strongest path from a to b, then, with the result of step #2, we get that the strongest paths have the following structure in the presence of a:



As a is the winner, a disqualifies b. Therefore, we get:

$$\min\{N[a,x_0]-N[x_0,a], P^{\text{old}}[x_0,b]\} > N[b,a]-N[a,b]. \tag{7}$$

As b is second best, b disqualifies x_0. Therefore, we get:

$$\min\{N[b,a]-N[a,b],\ N[a,x_0]-N[x_0,a]\} > P^{\text{old}}[x_0,b]. \tag{8}$$

But (7) and (8) are incompatible. Therefore, Munger's scenario is not possible under the Schulze method.

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Declarations

Conflict of interest The author declares no competing interests.

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